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Bellcomm

955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

date: June 17, 1971

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to: Distribution

from: D. A. Corey, R. C. Purkey, V. Thuraisamy

subject: The Predictability of a Collision Between the Skylab and a Piece of Orbital Debris - Case 610

ABSTRACT

Various previous studies have determined that the probability of a collision between the Skylab and a piece of orbiting debris is quite low -- on the order of .03%. Nevertheless, it seems prudent to take action to detect and avoid a collision if the effort required is not unreasonable and if there is a reasonable expectation of success.

Since NORAD already has the task of maintaining surveillance on objects in earth orbit and daily tracks and reports on their orbit parameters, they would seem to be the logical fundamental data source for collision prediction. Accordingly, NORAD personnel described their capabilities at a presentation at the Manned Spacecraft Center. NORAD would require additional facilities, personnel and funding to provide the detecting and predicting function and at best, they could predict the location of an object with an accuracy of about one kilometer. This memorandum shows that with this degree of accuracy, and given that a collision or near miss is predicted, the probability that one will actually occur is on the order of one to ten percent. This accuracy of prediction is certainly less than one would desire. Furthermore, there is evidence that there are a significant number of objects which represent a potential hazard to the Skylab but which are too small to be tracked by NORAD.

In summary, it appears that a significant increase in NORAD capability would be required to solve only a portion of the problem and even then, the solution would be of low quality and might simply produce many false alarms. Consequently, an active orbit collision detection and avoidance capability does not appear justified at this time -- especially in view of the rather low probability of a collision.

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MEMORANDUM FOR FILE

Introduction

Several recent studies have determined that the probability of a collision between the Skylab and a piece of orbital debris (including the spent Skylab S-II stage, the payload shroud panels, as well as assorted pieces from other programs) is small -- on the order of .03% (see References 1, 2, 3, and 4) for the current 235 nm Skylab orbit. Nevertheless, prudent operation dictates that action should be taken to detect and avoid a collision if the effort required is not unreasonable and if there is a reasonable expectation of success. Since NORAD already has the task of maintaining surveillance on objects in earth orbit and daily tracks and reports on their orbit parameters, they would seem to be the logical fundamental data source for collision prediction. Accordingly, NORAD described their capabilities to a number of MSC and MSC-contractor personnel at a briefing in December of 1970. As a result of the briefing and some work of their own, Messrs. M. E. Donahoo (MSC-MPAD) and J. Lewis (TRW Systems-Houston) have summarized the situation as follows:

1. The required increase on sensor (radar) tracking would increase the sensor workload by 20% to 50%.

2. NORAD would require two hours per day additional computer time to process the additional data.

3. The increase in NORAD orbital analyst activities would require at least one additional analyst per shift for the duration of the mission.

4. There is evidence that there are a significant number of objects which represent a potential

hazard to the Skylab, but which are too small to be tracked by NORAD.

5. The future position of a piece of debris that is tracked intensively is accurate to about one kilometer for reasonable prediction times.

Current NORAD equipment and funding are not capable of absorbing the extra load required by the first three points. This memorandum reports on a brief study which quantitatively assesses the impact of the fifth point. The fundamental question to be answered is: Given that a collision (or near miss) between the Skylab and some object is predicted to occur at some short time in the future -- what is the probability that a collision will actually occur?

Assumptions Used

To answer this question, we must know how much dispersion is associated with the predicted positions of the Skylab and the debris particle in the vicinity of the predicted collision. This information is contained in the covariance matrix which is customarily calculated as part of the radar data filter. The shape and size of the distribution depends heavily on the amount, quality, and time distribution of the radar measurements and the prediction time from the last data point to the potential collision instant. Regardless of the intrinsic accuracy of the radar measurement, the effect of increasing the prediction time is to spread the distribution along the orbit path, so that for predictions over several orbits, as would be required for collision avoidance, the distribution would become quite elongated.

Specific quantitative data on the potential accuracy of NORAD tracking is not readily available, so we are forced to use NASA MSFN data and assume that it is representative of what NORAD could do. Accordingly, a state vector uncertainty covariance matrix was selected from Reference 5. It represents the results attainable from two SL-1 passes over MSFN C-band (skin tracking) stations with the computed state vector propagated forward over four orbits. The size of its position dispersion corresponds to the value in item 5 above, assuming the one kilometer is a three-sigma value.

The matrix is presented in Table 1. It is expressed in UVW coordinates where U is along the object's radial direction, V is in the in-plane downrange direction, and W is out-of plane completing the orthogonal set. In order to save the reader the effort of taking square roots etc., the matrix is presented with the diagonal elements as the standard deviations (one sigma) in feet and feet per second and the off-diagonal terms are the correlation coefficients.

It was further assumed that the Skylab and the debris state vector uncertainties can each be represented by this matrix and that the matrix is valid at the time of predicted closest approach. Note that if the debris orbit plane intersects the Skylab orbit plane with some wedge angle, the UVW coordinate system for the two will not be parallel. Accordingly, an inertial coordinate system representation of the matrix will then be different for the Skylab and the debris.

In addition to the state vector uncertainties, the probability of a collision is also a function of the wedge angle between the orbit planes of the Skylab and the debris and the trajectory of the debris relative to the Skylab near the time of closest approach. Since the wedge angle can vary over a wide range (from 0to 90 degrees for an object with the same inclination as the Skylab), orbit plane wedge angle was simply treated parametrically without regard for the inclination of any particular piece of debris.

The in-plane relative trajectory characteristics depend on the apogee and perigee of the debris orbit relative to the Skylab's 235 nm circular orbit. An object with perigee at or just below 235 nm and apogee substantially greater than 235 nm for example, will pass the Skylab more or less horizontally. On the other hand, an object with a perigee altitude of 200 miles and an apogee altitude of 300 miles will have a more or less vertical motion relative to the Skylab. In either case, the radial component of velocity will be substantially smaller than the downrange component so the position error ellipsoid can be expected to have its long axis in the horizontal plane -- parallel (for the coplanar case) to the long axis of the Skylab error ellipsoid.

Consider for the moment, the case wherein the Skylab position is perfectly predictable and the location of the debris is known to the accuracy represented by the covariance matrix presented in Table 1. Further assume that the debris object is predicted to intercept the Skylab at some point in time and its orbit is coplanar with the Skylab. Near the time of predicted intercept, the volume of the Skylab can be envisioned to "sweep out" a volume of the object's position error ellipsoid. If that volume includes the actual location of the object, a collision will result. If the motion of the object relative to the Skylab is largely horizontal, the swept volume will be along the long axis of the ellipsoid whereas if the relative motion is largely vertical, the swept volume will be along the shorter vertical axis. The swept volume will be larger for the horizontal motion case and consequently the probability of a collision will be greater. Note that for the horizontal motion case, the length of the ellipsoid's long axis -- the uncertainty in downrange position -- will have no effect on the probability of a collision. Similarly, the size of the radial component of position uncertainty has no effect on the collision probability for the vertical relative motion case. The actual situation with Skylab location uncertainty as well as debris location uncertainty and relative motion near the time of closest approach different from strictly horizontal or vertical is more complicated than the simple case just described, but qualitatively, one can expect the same conclusion.

At the beginning of this study, it seemed reasonable to expect that the purely vertical and horizontal relative motion situations would be the limiting cases and one could bound the possible values of the probability of a collision by simply examining the two cases. Moreover, with relative motion occurring nearly along the eigenaxis of the position uncertainty ellipsoid, the problem appeared amenable to an analytic solution. Such a solution was found and is detailed in Appendix A. Basically, the technique works as follows: The Skylab is assumed to be represented by a rectangle oriented perpendicular to the line of its mean motion relative to the debris -- e.g. vertically or horizontally. As the Skylab moves through the debris position error ellipsoid, a parallelopiped shaped volume is swept out. The probability of a collision is then given by the product of the probability of the debris being in the swept volume times the probability that the Skylab is on the selected path -- i.e., the joint probability. (The rectangle shape for the Skylab was selected to simplify the integration limits in the computation of the probability that the debris is in the swept volume.) The total probability is then the integral of these joint probabilities over all the possible paths the Skylab might have (within the three sigma limits of its position uncertainty distribution). While the solution is analytical, its form is that of a multiple integral which was actually performed numerically. Note that only the position (upper left 3 x 3) portion of the uncertainty covariance matrix is considered in the solution. Incidentally, the simplifying assumption of relative motion along the eigenaxes was not required in this technique.

Since the analytic approach might have turned out to be more formidable than it did and it was necessary to verify that some of the simplifying assumptions required were justifiable, a second independent approach to the problem was also tried. This approach was a Monte Carlo trials technique wherein numerous random samples were taken from the covariance matrices representing the Skylab and debris state vector uncertainties. The samples were added to the mean state vectors and then propagated forward or backward in time to the point of closest approach. The closest approach distance was calculated and essentially an estimate of the miss distance cumulative probability distribution was constructed. Details of this technique are presented in Appendix B. Each approach has different strengths and weaknesses but fortunately, the results were comparable.

Results

Figures 1 and 2 present the cumulative distribution functions for the probability of the debris passing within some distance of the center of the Skylab given that the expected miss distance is zero -- that is, the means of the position uncertainty distributions intersect. Data for several different values of orbit plane wedge angle are included. Figure 1 represents the case wherein the relative motion is essentially horizontal, while Figure 2 represents the vertical relative motion case. With the Appendix B method, these cases were generated using debris orbits of 235 nm x 500 nm and 200 nm x 300 nm respectively (the numbers represent the altitudes of perigee and apogee). For the purposes of this discussion, a miss by less than 100 feet will be considered a collision and probability values will generally be related to this miss distance. The 100 foot figure includes an allowance for the size of the Skylab, the debris object, and a bit for conservatism. Probability values for other miss distances can be read from Figures 1 and 2.

As expected, the coplanar horizontal motion case results in the highest probability of a collision -- about 9%. When a small amount of wedge angle is present, however, the probability is substantially lower -about 1.2% for a ten degree wedge angle. The probability then increases monotonically with wedge angle to a maximum of about 9% again at the 180 degree wedge angle point. Note that the 180 degree wedge angle case is also a coplanar situation although the objects are converging from opposite directions.

For the vertical relative motion situation, Figure 2, the probability of the debris passing within 100 feet is about 1.3% for the zero wedge angle case. At ten degrees and larger wedge angles, the probabilities are virtually identical to the while the data for Figures 1, 2, and 3 was generated using the Appendix B technique. The slight differences are attributed to the fact that the Appendix B technique computes the probability associated with a 100 foot miss for example, on the basis of the debris coming within a 100 foot sphere of the Skylab. The Appendix A technique computes the 100 foot miss probability by using a square with an area equal to the area of a 100 foot radius circle. In fact, data points for all five figures were generated using both techniques and no significant discrepancies were found.

Conclusions

Given that a collision or near miss between the Skylab and a piece of orbital debris is predicted, the uncertainty of the prediction is so large that the probability that a collision will actually occur is on the order of 1 to 10%. The probability is principally a function of the wedge angle between the Skylab orbit plane and the debris orbit plane. It is smaller for wedge angles near 0 degrees and maximum for wedge angles near 180 degrees. For wedge angles near 0 degrees only, the probability is also a function of the debris apogee and perigee altitudes relative to the Skylab's 235 nm circular orbit. If the debris orbit has a perigee (apogee) altitude near 235 nm and an apogee (perigee) altitude significantly different from 235 nm, the probability of a collision, given that one is predicted, is near the 10% value. Otherwise, it will be nearer the 1% value. In addition, of course, the probability is a function of the predicted miss distance. The probability decreases rather rapidly for increasing predicted misses in the out-of-plane or radial directions but it is significantly less sensitive to predicted misses in the in-plane downrange direction.

These probability values are associated with tracking and prediction accuracies which NORAD has stated are about the best they can do for predicting Skylab-debris orbital collisions. Even this level of accuracy would require NORAD facilities, personnel, and funding in excess of what is currently available. The accuracy of the predictability of a collision is certainly less than one would desire and an active orbit collision prediction and avoidance capability does not appear justified at this time -- especially in view of the rather low probability of a collision.

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Attachments

horizontal relative motion case. This identity occurs because the few hundred feet per second relative radial velocity (or difference in velocity magnitude) is small compared to the thousands of feet per second relative out-of-plane velocity. In the short period of time during which the position uncertainty error ellipsoids intersect, relatively little vertical motion takes place.

Figure 3 presents the 100 foot miss distance probabilities versus wedge angle for both cases. The Figure 3 data repeats data shown on Figures 1 and 2 but was generated and presented to show that the probabilities are symmetric about zero degrees wedge angle as one would expect.

Figures 4 and 5 present the probabilities of a collision given that the debris is predicted to miss the Skylab by some amount. Figure 4 considers the horizontal coplanar relative motion case, while Figure 5 presents the vertical coplanar relative motion case. Note that, as one might expect, the rate of decrease in the probabilities with increasing predicted miss distance (displacement of the means) varies according to the direction of the predicted miss and this variation correlates with the relative size of the one sigma position errors for that direction. In the horizontal relative motion case, the value of the probability of a collision is an order of magnitude lower than the zero miss value for a predicted miss of about 550 feet in the radial direction or about 350 feet in the out-of-plane direction. (Recall that in the horizontal coplanar motion case, a downrange displacement of the point of closest approach has no effect on the probabilities.) The probability of a collision in the vertical coplanar relative motion case is down an order of magnitude from the zero miss value when the predicted out-of-plane miss is about 400 feet or when the predicted downrange miss is about 3000 feet.

The probability of a collision given a non-zero mean miss distance is a function of the orbit plane wedge angle just as it was in the zero mean miss distance case. In fact the functional relationship is of the same form as shown in Figure 3 where the maximum probability points occur at the 0 and 180 degree points and minimum values occur near ten degrees wedge angle. The amount of data required for detailed plots illustrating this fact were not generated although sufficient data was generated to verify its truth.

A careful reader will note a slight discrepancy between zero miss distance probability values presented on Figures 4 and 5 and those presented on Figures 1, 2, and 3. The data for Figures 4 and 5 was computed using the Appendix A technique

REFERENCES

1. Skylab/S-II Stage Recontact Study, Prepared for NASA/MSFC by Northrop Corp., Huntsville, Alabama; TR-795-888, D. L. Cobb, B. L. Spencer, J. L. Moorhead, W. G. Greenleaf, February 1971.

2. Skylab/Shroud Recontact Analysis, Prepared for NASA/MSFC by Northrop Corp., Huntsville, Alabama; TR-795-805, J. L. Moorhead, W. G. Greenleaf, D. L. Cobb, B. L. Spencer, November 1970.

3. Comparison of the MSC and MSFC Collision Probability Results of the Skylab With Objects in Earth Orbit, NASA/MSC/MPAD Memorandum 70-FM37-134, H. E. Donahoo, October 1970.

4. Additions and Corrections to Skylab/Satellite Collision Probabilities, NASA/MSFC, S&E-AERO-MM-35-70, August 21, 1970.

5. MSFN Support for the Skylab 1/2 Rendezvous, NASA/MSC Memorandum 70-FM46-152, June 3, 1970.

TABLE 1

STATE VECTOR UNCERTAINTY COVARIANCE MATRIX

	U	V	W	$\dot{U}$	$\dot{V}$	$\dot{W}$
U	205.597	.411506	.1120334	-.466978	-.872471	.589533
V		1020.142	.41737	-.995976	-.3505265	.6641635
W			135.6750	-.422565	.023577	.423162
$\dot{U}$				1.091279	.410389	-.705177
$\dot{V}$					.255555	-.517395
$\dot{W}$						.3707465

Note - Diagonal elements are the standard deviations, in feet and feet/second units, off-diagonal elements are the correlation coefficients.

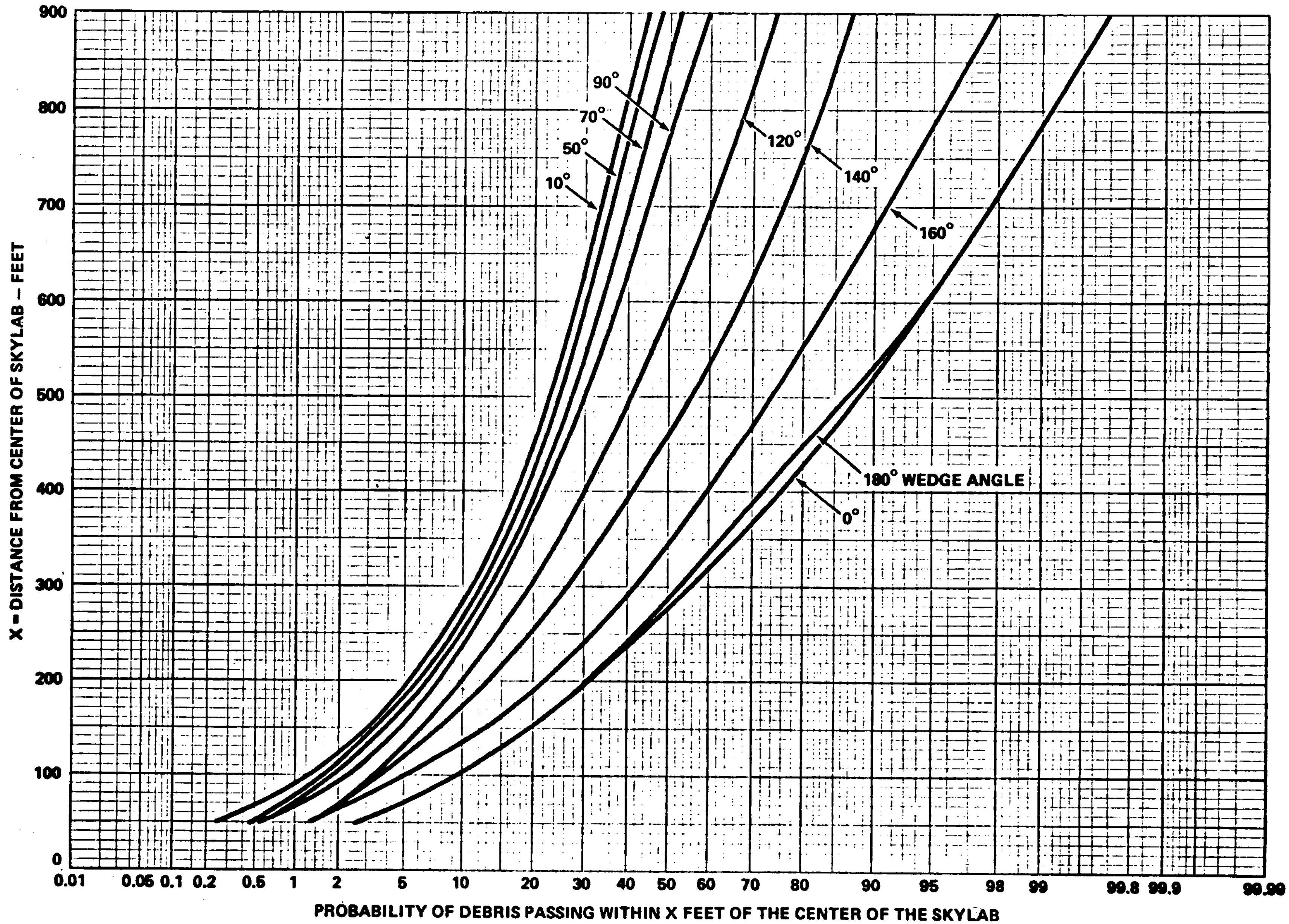

FIGURE 1 - CUMULATIVE DISTRIBUTION FUNCTION OF DEBRIS MISS DISTANCE FOR VARIOUS ORBIT PLANE WEDGE ANGLES;HORIZONTAL RELATIVE MOTION (DEBRIS IN 235 NM x 600 NM ORBIT) ZERO MEAN MISS DISTANCE

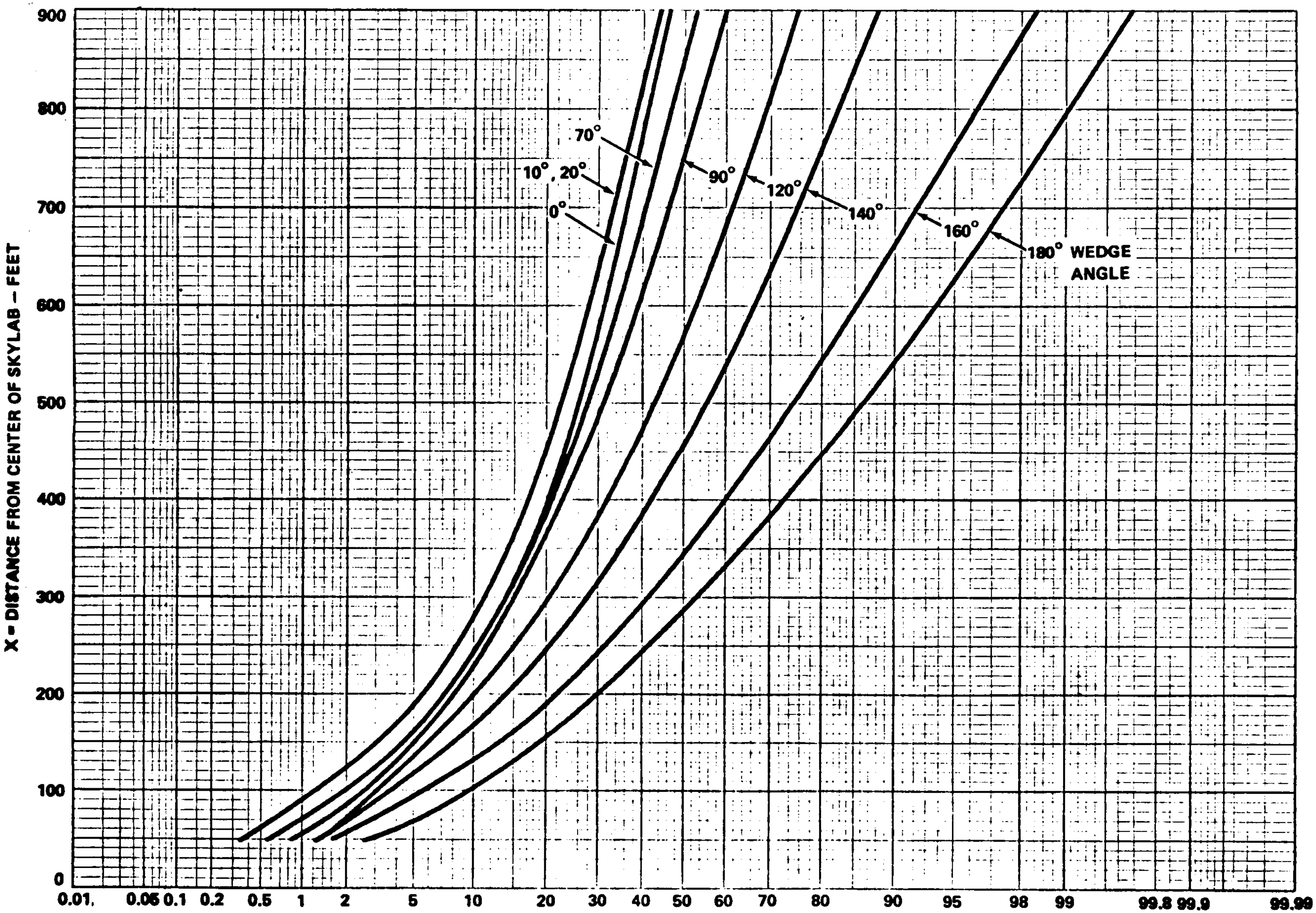

FIGURE 2 - CUMULATIVE DISTRIBUTION FUNCTION OF DEBRIS MISS DISTANCE FOR VARIOUS ORBIT PLANE WEDGE ANGLES; VERTICAL RELATIVE MOTION (DEBRIS IN 200 NM x 300 NM ORBIT) ZERO MEAN MISS DISTANCE

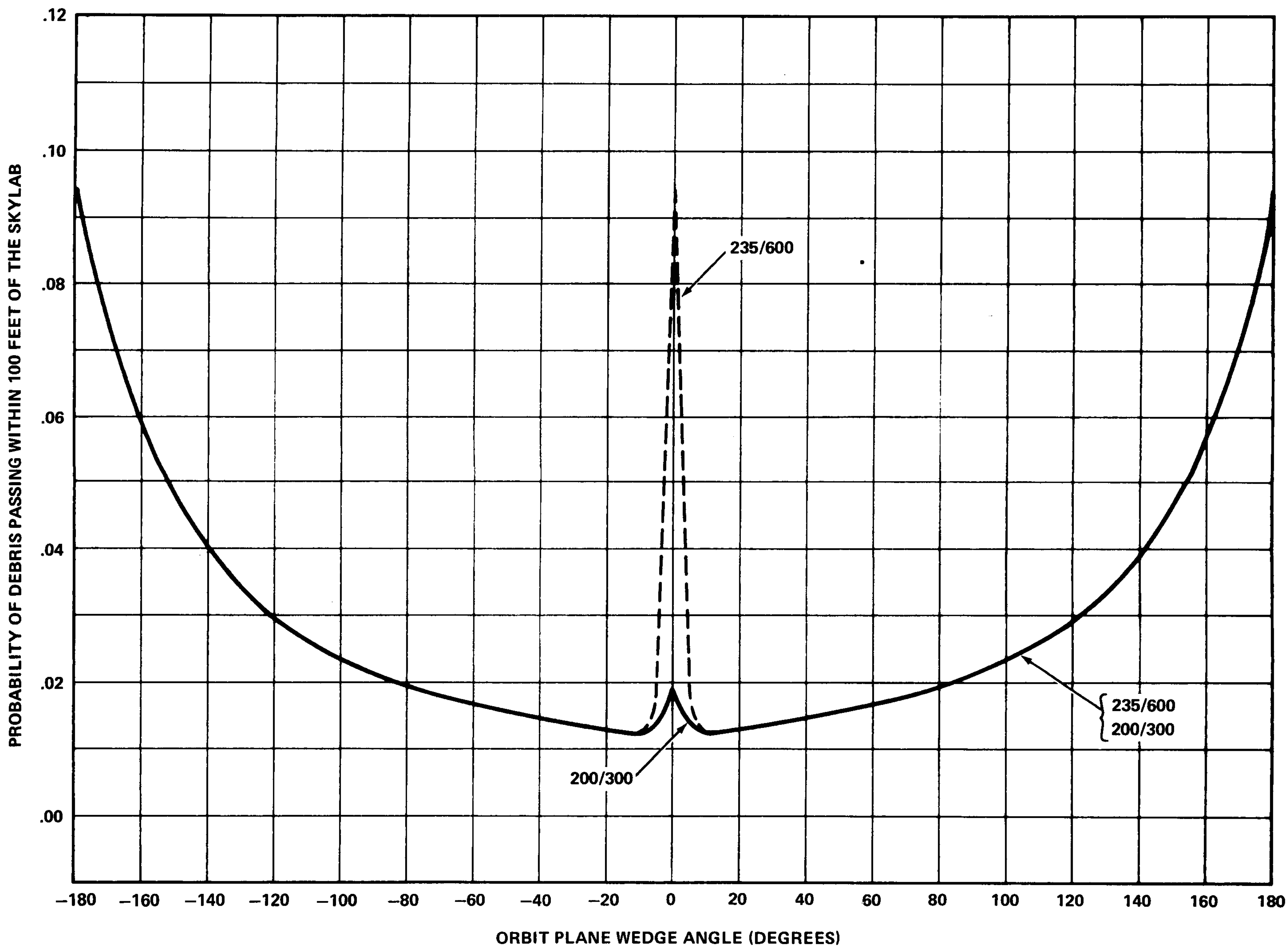

FIGURE 3 - PROBABILITY OF DEBRIS MISSING BY LESS THAN 100 FEET VERSUS ORBIT PLANE WEDGE ANGLE; ZERO MEAN MISS DISTANCE

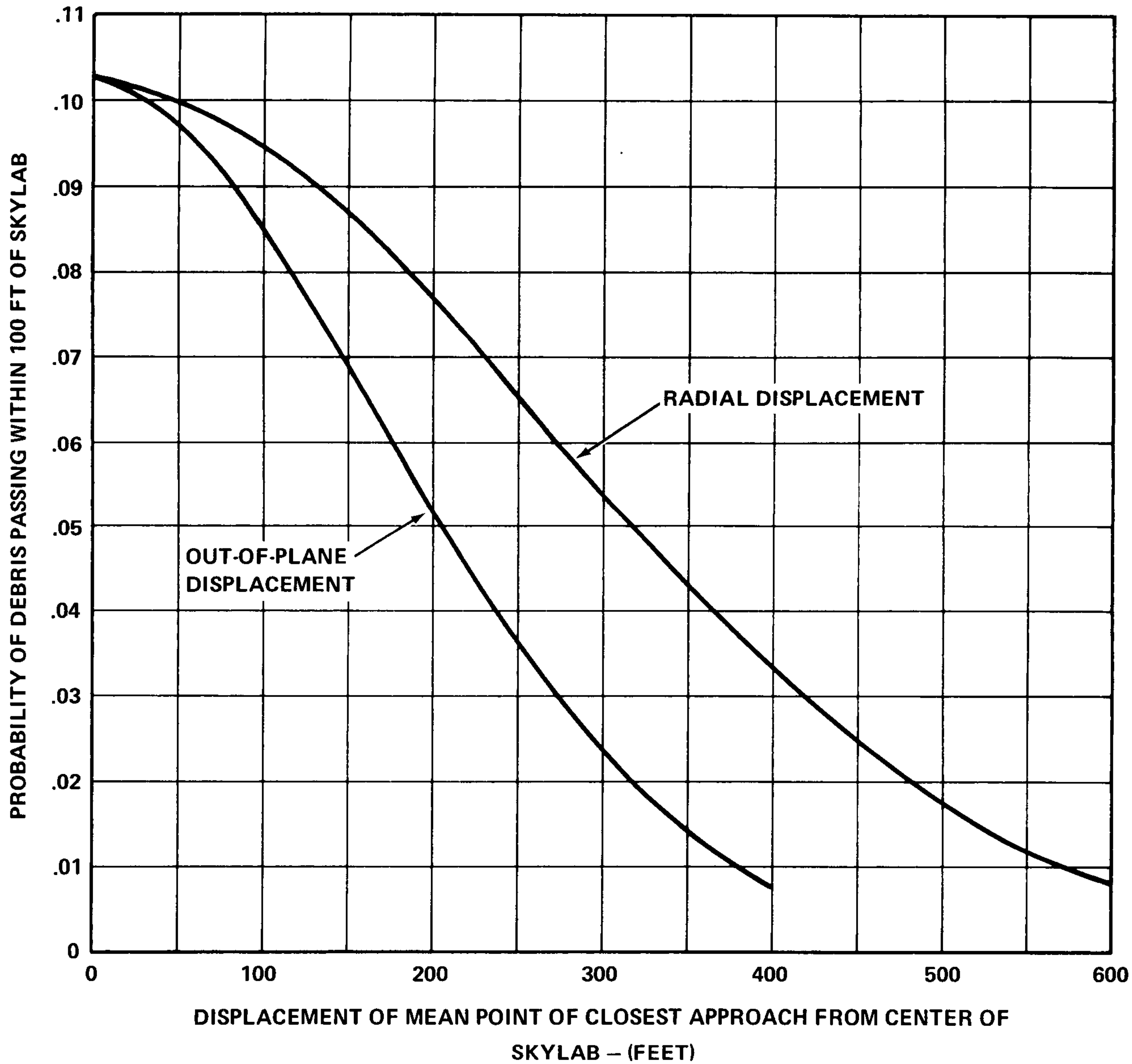

FIGURE 4 - PROBABILITY OF A COLLISION BETWEEN DEBRIS AND SKYLAB VS DISPLACEMENT OF THE MEAN POINT OF CLOSEST APPROACH; HORIZONTAL COPLANAR RELATIVE MOTION

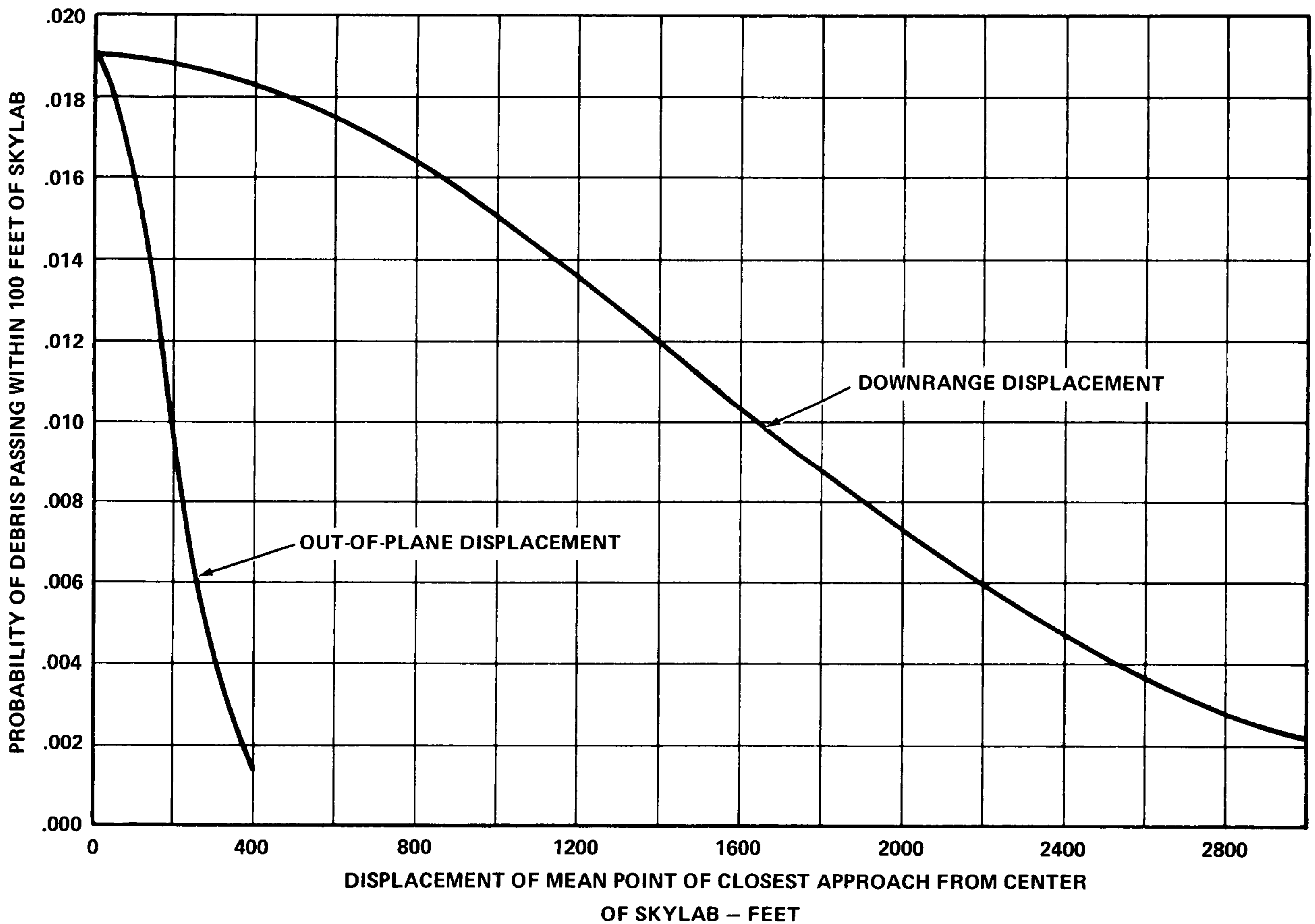

FIGURE 5 - PROBABILITY OF A COLLISION BETWEEN DEBRIS AND SKYLAB VS DISPLACEMENT OF THE MEAN POINT OF CLOSEST APPROACH; VERTICAL CO-PLANAR RELATIVE MOTION

APPENDIX A

ANALYTIC DETERMINATION OF THE PROBABILITY OF A COLLISION

A possible collision between Skylab and a piece of orbiting debris must be expressed in probabilistic terms. Actual locations of both objects are predictable from tracking data only in a statistical sense by a three-variable Gaussian distribution function. Since the locus of a set of equiprobable points in such a distribution is an ellipsoid, the region in which the object is likely to be located is often called the uncertainty ellipsoid. Uncertainty ellipsoids move with time along the mean or expected paths of the Skylab and the debris, which are assumed to correspond to the predicted paths. If the uncertainty ellipsoids (say for the three sigma probability level) do not intersect at any time, then it is known that the probability of a collision is negligible. If, on the other hand, these ellipsoids do intersect for some duration of time, then a collision is possible. The statement that a collision or near miss is predicted then means that at some point in time, the centers of the two uncertainty ellipsoids coincide or nearly coincide.

The debris (D) is represented by a point and the Skylab (L) is represented by a rectangle oriented perpendicular to the direction of the relative velocity of L with respect to D. It is convenient to take rectangular coordinate axes with the origin at the center of the D-ellipsoid and the Z-axis parallel to the velocity of L relative to D. The X and Y axes are parallel to the sides of the rectangle that represents L. The implicit assumption of rectilinear motion here is reasonable since we are concerned only with the portions of the trajectories when the ellipsoids of uncertainty intersect. Consider another, small rectangle

$$R = \{(x, x + \Delta x) \times (y, y + \Delta y)\}$$

in the (x,y) plane. Since the relative velocity of the means is perpendicular to this plane, it is valid to ask: what is the probability P_R that the center of L passes through R? An equivalent question is what is the probability that at any instant in time, the center of L lies in the doubly infinite tube with cross-section R? Taking this instant in time to be when the center of L-ellipsoid is on the (x,y) plane,

$$P_R = \Delta x \Delta y \int_{-\infty}^{\infty} f(x,y,z)\, dz \qquad (1)$$

where f(x,y,z) is the joint normal probability function associated with L.

The joint normal density function for random variables X, Y, Z, is given by

$$f(x,y,z) = \frac{e^{-\frac{1}{2} u^T C^{-1} u}}{(2\pi)^{3/2} \sqrt{\det C}} \qquad (2)$$

where

$u^T = \{x-m_1,\ y-m_2,\ z-m_3\}$,

C is the covariance (moment) matrix, and

m_1, m_2, m_3 are the coordinates of the mean.

If the covariance matrix is diagonal, the axes of the equi-probable ellipsoids are parallel with the axes of reference. The square roots of the diagonal elements of C are the standard deviations, σ_x, σ_y, σ_z, of the random variables.

The integral

$$\int_{-\infty}^{\infty} f\ (x,y,z)\ dz$$

in Equation (1) is precisely the density function $f_z(x,y)$ for the marginal distribution of X and Y. It is known that $f_z(x,y)$ itself corresponds to a normal distribution in the two variables X and Y with standard deviations, σ_x and σ_y. Thus, P_R is given in terms of a two-dimensional distribution,

$$P_R = f_z\,(x,y)\ \Delta x \Delta y \quad . \tag{3}$$

Let g(x,y,z) be the density function for D. The probability P_{RC} of collision while the center of L lies in R is then the probability of finding D in the doubly infinite tube with rectangular cross-section

$$\{(x-a,\ x + \Delta x + a)\ \text{by}\ (y-b,\ y + \Delta y + b)\} \quad .$$

Here 2a, 2b are the sides of the rectangular cross-section of L. Thus, P_{RC} is given by the product

$$P_{RC} = P_R \cdot \int_{z'=-\infty}^{\infty} \int_{x'=x-a}^{x+a} \int_{y'=y-b}^{y+b} g(x',\ y',\ z')\ dz'dx'dy' \tag{4}$$

Again,

$$\int_{z'=-\infty}^{\infty} g(x',\ y',\ z')\ dz' = g_z\ (x',\ y') \quad . \tag{5}$$

Thus,

$$P_{RC} = P_R \cdot \int_{x'=x-a}^{y+a} \int_{y'=y-b}^{y+b} g_z\,(x',\, y')\; dx'\; dy' \tag{6}$$

$$+ P_R \cdot e_R \quad ,$$

where the error term $P_R \cdot e_R \rightarrow 0$, as Δx, $\Delta y \rightarrow 0$.

Finally, the probability P of a collision is

$$P = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \left(f_z(x,y) \int_{x'=x-a}^{x+a} \int_{y'=y-b}^{y+b} g_z\,(x',\, y')\; dx'dy' \right) dxdy \tag{7}$$

Thus the computation of P has been reduced to the evaluation of a four-fold multiple integral where the integrand is the product of two two-dimensional marginal density functions. It can be shown that (Reference A-1) the marginal density function

$$f_z\,(x,y) = \frac{e^{-Q_z(x,y)/2}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \quad , \tag{8}$$

where

$$Q_z(x,y) = \frac{1}{\sqrt{1-\rho^2}} \left(\frac{(x-m_1)^2}{\sigma_1^{\,2}} + \frac{(y-m_1)^2}{\sigma_2^{\,2}} - \frac{2\rho\,(x-m_1)\,(y-m_2)}{\sigma_1\sigma_2} \right) \; .$$

ρ is the correlation coefficient for the two-dimensional matrix obtained from the first two rows and columns of the covariance matrix. Similar expressions are obtained for $f_x(y,z)$ and $f_y(x,z)$.

The integrand in Equation (7) written out fully is as follows:

$$\frac{e^{\left[\frac{\frac{(x-m_1)^2}{\sigma_1^2} + \frac{2\rho(x-m_1)(y-m_2)}{\sigma_1\sigma_2} - \frac{(y-m_2)^2}{\sigma_2^2}}{2(1-\rho^2)}\right]} \cdot e^{\left[\frac{-\frac{x'^2}{\sigma_1'^2} + \frac{2\rho' x' y'}{\sigma_1'\sigma_2'} - \frac{y'^2}{\sigma_2'^2}}{2(1-\rho'^2)}\right]}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2} \cdot 2\pi\sigma_1'\sigma_2'\sqrt{1-\rho'^2}}$$

The primed quantities refer to the debris (D) and the unprimed to the Skylab (L).

To carry out the integration in (7), an Algol program was developed. The basic algorithm merely uses Simpson's three-eighths rule, but the Algol language permits an indirect recursive device that greatly simplifies the program for multi-dimensional integration (see Reference A-2).

Completely arbitrary orientations of the two ellipsoids as well as non-intersecting mean paths can be handled in the fashion outlined above. Actual computations, however, were carried out for the cases that have physical significance. Since any general case may be obtained from some simple case by a translation and/or rotation of the L-ellipsoid -- which means that the covariance matrix is pre- and post-multiplied by appropriate translation and/or rotation matrices -- the computation of the collision probability in the most general case is only slightly more cumbersome.

APPENDIX A REFERENCES

A-1. H. Cramer, Mathematical Methods of Statistics, Princeton, 1961.

A-2. Univac 1106-1108 Algol, Appendix E, UP-7544, Rev. 1.

APPENDIX B

A Monte Carlo trials approach was also used to determine the probabilities of a collision. This technique involves four steps:

1. Constructing example case mean state vectors,
2. Sampling from the uncertainty covariance matrices and adding the sample deviation to the mean state vector,
3. Computing the distance at the point of closest approach, and
4. Computing the probability values as a function of miss distance.

The Skylab mean state vector assumes a 235 nm altitude circular orbit. The debris object mean position state vector was set up to have the desired position relative to the Skylab (e.g. the same position vector for the zero mean miss case) and the mean velocity vector is determined on the basis of the desired apogee and perigee radius and orbit plane wedge angle.

The second step involved computing a sample position and velocity vector for each state. This was done by first taking a random sample deviation vector from the covariance matrix for each of the two states. Since the covariance matrix was assumed to apply to the local vertical coordinate system of each state, the deviation vectors were each rotated to the inertial coordinate system in which the nominal (or mean) states were defined. The rotated deviation vectors were then added to the mean or nominal state vectors to obtain the sample state vectors. The set of random numbers used to sample the covariance matrix is non-repeating for the set of trials used for each case; however, the same set of random numbers was used for every case studied.

With the sample state vectors set up, the point of closest approach was then computed. At first this was done by iteratively propagating the states using Keplerian motion until the point of closest approach was located. This was found to be a slow and unnecessary approach. The technique used to compute the data presented in this memorandum assumed that the point of closest approach occurs along a line defined by the Skylab sample velocity vector minus the debris sample velocity vector and originating at the debris sample position. The point of closest approach is found by dropping a perpendicular from the Skylab sample position to this velocity difference vector. This technique was found to produce answers identical to those obtained using Keplerian propagation, though an order of magnitude less computer time was required.

For each case studied, 1500 trials were run and the number of times the miss distance was less than some distance (e.g. 100, 200, 300 feet, etc.) was counted. These values were then simply divided by 1500 to provide an estimate of the probability of the debris passing within the given distance of the center of the Skylab. The probability levels were then plotted as a function of miss distance to provide an estimate of the cumulative distribution function for each case. Figures 1 and 2 are examples of these plots though other cases were run. Data points corresponding to the data on Figures 4 and 5 were also computed with this technique as a cross check between the two techniques.

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E. D. Geissler/S&E-AERO-DIR
C. C. Hagood/S&E-CSE-A
O. M. Hardage/S&E-AERO-MF
G. B. Hardy/PM-SL-EI
J. P. Lindberg/S&E-AERO-M
B. S. Perrine, Jr./S&E-AERO-MMD
R. E. Tinius/S&E-CSE-MP

TRW - Houston

J. Lewis

Bellcomm

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